05/01/21

**THM 121 Business Mathematics**

**Final Exam**

**Note to the students**:

* Calculations to reach your answers shall be thoroughly shown. Otherwise, questions will NOT be graded.
* You can use a calculator throughout the exam.
1. Consider the following equation of a line: 4χ + 3y = 5
2. Find the equation of the line that passes through the point (-2, 3) and is **perpendicular** to the line 4χ + 3y = 5. (**3** Points)

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1. Sketch the **graph** of the line found in part a). (**2** Points)
2. A certain stock had an initial public offering (IPO) price of **$ 10 per share** and is traded 24 hours a day. Sketch the graph of the share price over a 2-year period for each of the following cases:
3. The price **increases** steadily to **$ 50** *over the first 18 months* and then **decreases** steadily to **$ 25** per share *over the next 6 months*. (**2** Points)
4. The price takes just *2 months* to **rise** at a constant rate to **$ 15** a share and then slowly **drops** to **$ 8** *over the next 9 months* before steadily **rising** to **$ 20**. (**2** Points)
5. The price steadily **rises** to **$ 60** a share *during the first year*, at which time an accounting scandal is uncovered. The price **goes down** to **$ 25** a share and then steadily **decreases** to **$ 5** over the next 3 months before **rising** at a constant rate to close at **$ 12** at the end of the 2-year period. (**3** Points)

3) Farmers can get **$ 8** per bushel for their potatoes on July 1, and after that, the price drops by **5** cents per bushel per day. On July 1, a farmer has **140 bushels** of potatoes in the field and estimates that the crop is **increasing** at the rate of 1 bushel per day.

a) Express the farmer’s **revenue** from the sales of the potatoes as a function of the **time** at which the crop is harvested. (**2** Points)

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b) **Sketch** the graph of the function found in part a). (**3** Points)

1. Estimate **when** the farmer should harvest the potatoes to **maximize revenue**. (**2** Points)

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1. Producers will supply χ units of a certain commodity to the market when the price is *p* = *S*(χ) dollars per unit, and consumers will demand (i.e. buy) χ units when the price is *p* = *D*(χ) dollars per unit, where

***S*(χ) = 3χ + 25 and *D*(χ) = 415 / (2χ + 1)**

1. Find the **equilibrium production level** χ, and the **equilibrium price** *pe*. (**2** Point)

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1. Draw the **supply** and **demand** curves on the same graph. (**3** Points)
2. Where does the supply curve **cross the y axis**? Describe the **economic significance** of this point. (**2** Points)

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1. Studies indicate that *t* years from now, the population of a certain country will be *p* = 0.2*t* + 1,500 thousand people, and that the gross earnings of the country will be *E* million dollars, where

***E (t*) =** $\sqrt{(9t^{2}+0.5t+179)}$

1. Express the per capita earnings of the country **P** = E / P as a function of time t. (Take care with the units) (**2** Points)

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1. Draw the graph of per capita earnings function of time? (**3** Points)
2. What happens to the **per capita earnings** in the long run (i.e. as ***t*** → **∞**)? (**2** Points)

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1. If the air temperature on a given day is **80°F**, the heat index *I*(*h*) (also in °F) can be approximated by the following piecewise function, where *h* is the relative humidity as a percentage:

***I* (*h*) =** $\left\{\begin{array}{c}80 if 0\leq h\leq 40\\80+0.1\left(h-40\right) if 40<h\leq 80\\0.005h^{2}-0.65h+104 if 80<h\leq 100\end{array}\right.$

1. Sketch the graph of I(*h*)? (**3** Points)
2. What **relative humidity** produces a heat index of **83°F**? (**1** Point)

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1. Is the heat index function I(*h*) **continuous** at *h* = 40? What about at *h* = 80? (**3** Points)

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**N.B**. Round your answers to the **nearest cent** for questions 2, 3, 4, 5 & 6.

**GOOD LUCK!**